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It should be added that all annuities are to be valued, as formerly with formula (1), at the rate of interest i/m. Formulas (2) and (3) prove very easy to apply in a concrete problem and the values of the annuities involved are easily determined. We shall conclude with a typical though simple example. Let it be desired to find the value or price of a loan of \$50,000 offered at 6 per cent. and dated September 1, 1914; the principal to be repaid in fifteen installments, \$2,000 September 1, each year from 1919 to 1928 and \$6,000 September 1, from 1929 to 1933, to net the purchaser 5 per cent.

Here we have $f = 5$, $m = t = 1$, $g = .06$, $i = .05$ and $n = 2$ and we can use formula (3) as simplified and expressed in formula (4), whence

$$k = \left(1 - \frac{a(2)\cdot 6000 + a(1)(2000 - 6000) - a(0)\cdot 2000}{50{,}000 a_{\overline{1}|}}\right)\left(\frac{.06 - .05}{.05}\right),$$

where

$$a(2) = a_{\overline{m(f+r_1t+r_2t)}|} = a_{\overline{20}|}$$

$$a(1) = a_{\overline{m(f+r_1t)}|} = a_{\overline{15}|}$$

$$a(0) = a_{\overline{mf}|} = a_{\overline{5}|},$$ all to be valued at 5 per cent.

The values of these annuities can be obtained from any set of tables. Hence, finally, the premium is $k = .0966977$, and the price of the entire loan becomes \$50,000 $(1 + k)$ or \$54,834.89.

A GEOMETRICAL INTERPRETATION OF GREEN'S FORMULA.

By W. V. LOVITT, Purdue University.

It is the object of this note to give a geometrical interpretation of Green's formula:

$$\int_{(C)} P(x, y)dx + Q(x, y)dy = \int_A\int\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy.$$

The single integral is to be taken in the positive direction around a closed curve

$$(C) \qquad C(x, y) = 0$$

in the xy-plane and the double integral over the interior A of C. To derive this formula it is sufficient to prove the formulas

$$(1) \qquad \int_A\int \frac{\partial P}{\partial y}dxdy = -\int_{(C)} Pdx,$$

$$(2) \qquad \int_A\int \frac{\partial Q}{\partial x}dxdy = \int_{(C)} Qdy$$

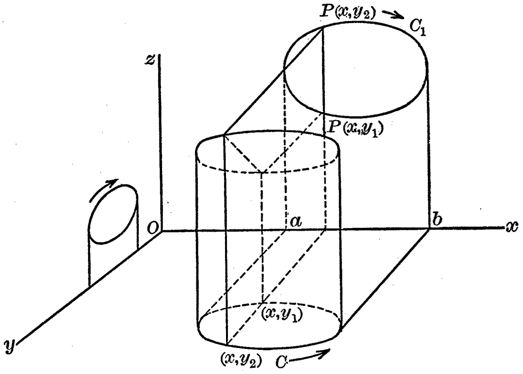
(see for example Goursat-Hedrick, § 126), and for simplicity it may be assumed that A is a region of the xy-plane in which $\partial P/\partial y$ and $\partial Q/\partial x$ are of constant sign. We will discuss equation (1) and leave equation (2) for the consideration of the reader.

Construct the surface

$$(P) \qquad z = P(x, y).$$

The cylinder $C(x, y) = 0$ intersects the surface (P) in a curve the projection of which on the xz-plane we designate by C_1. Let the two parts of C be represented by

$$y = y_1(x), \qquad y = y_2(x)$$



as in the figure. Then the two parts of C_1 are

$$z = P(x, y_1), \qquad z = P(x, y_2)$$

and it is clear that the integral

$$\int_{(C)} P dx = \int_{(C_1)} z dx$$

represents the area of C_1. The double integral may be written

$$\int_A\int \frac{\partial P}{\partial y} dxdy = \int_A\int \frac{\dfrac{\partial P}{\partial y}}{\sqrt{\left(\dfrac{\partial P}{\partial x}\right)^2 + \left(\dfrac{\partial P}{\partial y}\right)^2 + 1}} \sqrt{\left(\frac{\partial P}{\partial x}\right)^2 + \left(\frac{\partial P}{\partial y}\right)^2 + 1}\, dxdy$$

$$= \int_A\int \cos \beta dS,$$

where dS is the element of the surface (P) and β is the angle between the normal to (P) and the y-axis. This shows that the double integral is also an expression for the area of C_1. It is now evident that the two sides of the equation (1) have the same absolute value. There remains the question of sign.

A curve in the xy-plane will be said to be traversed in the positive sense when it is gone over in the direction indicated by the arrow in the figure. For the xz-plane the positive direction is taken as opposite to that indicated by the arrow. For the yz-plane it will be found necessary in the discussion of equation (2) to take the positive direction as indicated by the arrow in the figure. Areas are to be considered positive when their boundaries are traversed in the positive sense and negative when their boundaries are traversed in the negative sense. These definitions of the positive sense for the three coordinate planes are independent of the octants and seem to be the simplest for our present purpose.

By an inspection of the figure we see that the curves C and C_1 are traced by corresponding points in the same sense when $\partial P/\partial y < 0$ and in opposite senses when $\partial P/\partial y > 0$. If then we take into account the sign of the projective factor $\partial P/\partial y$ and the sense in which the curve C_1 is traversed when the curve C is gone over in the positive sense, we find that always

$$-\int_{(C)} P dx = \int_A \int \frac{\partial P}{\partial y} dx dy.$$

ON A CERTAIN CLASS OF DETERMINANTS.

By ERNESTO PASCAL, Milan, Italy.

Translated by permission from the *Rendiconti della R. Accademia delle Scienze Fisiche e Matematiche di Napoli* (3), volume XX, 1914.

The AMERICAN MATHEMATICAL MONTHLY for June, 1914, volume 21, page 184, contains the following question under the heading "A simple algebraic paradox."

Given two linear homogeneous complex equations

$$(1) \qquad \begin{aligned} (a + bi)(p + qi) + (c + di)(r + si) &= 0, \\ (a' + b'i)(p' + q'i) + (c' + d'i)(r' + s'i) &= 0. \end{aligned}$$

In order that these should be compatible it is necessary and sufficient that

$$\begin{vmatrix} (a + bi) & (c + di) \\ (a' + b'i) & (c' + d'i) \end{vmatrix} = 0.$$

This complex equation is equivalent to the *two* real equations

$$(1) \qquad \begin{aligned} ac' + a'c &= bd' - b'd, \\ ad' + bc' &= a'd + b'c. \end{aligned}$$

Both of these must be fulfilled if (1) is to subsist. On the other hand equations (1) are equivalent to

$$\begin{aligned} ap - bq + cr - ds &= 0, \\ bp + aq + dr + cs &= 0, \\ a'p - b'q + c'r - d's &= 0, \\ b'p + a'q + d'r + c's &= 0. \end{aligned}$$